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A STARTING RULE FOR DATA COLLECTION IN QUEUEING SIMULATIONS.(U)

AUG 79 V G ADLAKHA, G S FISHMAN N00014-76-C-0302

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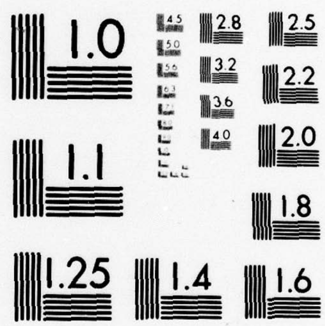
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(12) LEVEL II

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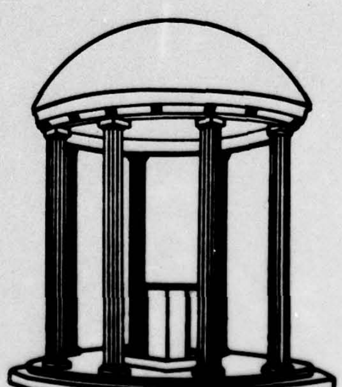

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(12) LEVEL II

(6) A STARTING RULE FOR DATA COLLECTION

IN

QUEUEING SIMULATIONS.

(10) Veena G./Adlakha ~~and~~ George S./Fishman

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ABSTRACT

This paper proposes a rule for determining when to start collecting data in a queueing simulation. The rule is designed to reduce dependence between the empty (queue) and idle (servers) *initial conditions* and the collected sample record. The rule is an outgrowth of earlier work by Fishman and Moore (1978) and relies on a comparison between *a priori* information on the activity level (traffic intensity) and a corresponding sample estimate computed during the course of simulation. Experiments with simulations of the M/M/c queue with $c = 1,2,4$ and $\rho = .7,.8,.9,.95$ reveal that the rule reduces and in most cases removes the dependence on the empty and idle initial conditions. In particular, the rule begins data collection when the simulation is in a congested state or in the steady state. The rule is well behaved in that it has low probabilities of requiring long runs before data collection is started. Although our data suggests an association between the rule's performance and activity level, the performance is insensitive to variation in the number of servers. Since the rule is based upon the activity level, a parameter that frequently can be computed from the input parameters of the simulation, the rule is easily generalized to a wider class of queueing simulations. A subsequent study (Adlakha and Fishman 1979) demonstrates the appeal of the starting rule when used with a proposed stopping rule for computing interval estimates of parameters of interest.

1. Introduction

This paper proposes a rule for starting data collection in a queueing simulation. The rule is an outgrowth of work by Fishman and Moore (1978) who described a framework for research into the problem of reducing the effect of initial conditions on sample records collected for statistical analysis in a discrete event simulation. *Initial Conditions* refer to the states of critical variables at the beginning of a simulation run. Because of the dependence among phenomena in a simulation and the temporal nature of much of this dependence, the choice of initial conditions influences the observed time paths of these phenomena. The suggested framework in the Fishman and Moore paper encourages the use of *ancillary* information such as the theoretical activity level in a queueing simulation to provide guidance as to when data collection for analysis should begin in a simulation. In particular, if a comparison between the theoretical and sample activity levels in a simulation shows that certain conditions are met, empirical evidence in their paper shows that the queueing simulation is in a congested state for an important range of theoretical activity levels. Therefore, data collection initiated at that point reflects initial conditions for a congested system rather than for an undercongested system, as would occur if data collection starts at the beginning of a simulation whose initial conditions include all idle servers and empty queues. Since most simulators are inclined to accept upward bias due to initial congestion more readily than downward bias due to initial undercongestion, this new-found ability to induce congestion is a major step forward in alleviating the often asserted

problem of initial conditions: When does a simulation achieve representative state behavior of the system being studied?

Ideally, one would like to begin data collection when the system is in the steady state. However, the very act of using information on the sample path up to the point at which a decision is to be made about starting data collection from that point onward necessarily creates a conditionality and, therefore, prevents a determination of when the steady state arises in a given run. Moreover, from a probabilists's point of view one regards the steady state as a relative, rather than an absolute, concept that is achieved as a limiting operation. In support of these assertions, the empirical evidence in the Fishman and Moore paper shows that the steady state is not a *point of attraction* for the type of data-based rules they employ. However, the inducement of congestion is possible.

Fishman and Moore presented an explicit rule for achieving this congestion.† Although the rule performed well, the highly variable data collection starting times to which it led allowed for the possibility of excessive cost before achieving congestion. Their report concludes with a recommendation that further search of rules to induce congestion should consider an *iterative* rule which they describe. This paper presents the results of a considerably more elaborate sampling experiment designed to evaluate this rule, hereafter referred to as Rule 1. By way of summary, the results to be presented here show that the candidate rule can induce congestion with considerably less risk of excessive cost for a variety of queueing models and activity levels.

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†The rule is called Rule 2 in their paper.

Section 2 formulates the problem and Section 3 describes the experimental design and measures of evaluation. Section 4 presents an analysis of the empirical results and recommends specific values for the parameters of the rule. Section 5 presents an analysis of the distribution of starting times and shows that the new recommended rule leads to considerably less skewness than the rule recommended in Fishman and Moore (1978). The implications for cost are discussed in detail. Section 6 draws overall conclusions and describes a proposed algorithm for implementing the rule.

Before beginning the technical discussion, we wish to stress two points. Firstly, the ancillary information used for decision making here is the activity level of the queueing system being simulated. We contend that this quantity can frequently be computed from the input parameters of the simulation. This computation is also possible in many, though not all, large scale queueing simulations. Although we recognize that our results demonstrate the value of the proposed starting rule with relatively simple systems, we feel that its applicability with perhaps some modification to larger scale systems is a possibility that a thoughtful simulation user would want to consider seriously.

The second point concerns the ultimate purpose of starting data collection in the steady state or in a congested state. In a second paper (Adlakha and Fishman 1979), we develop a stopping rule for data collection which when used with the starting rule proposed here produces confidence intervals via the autoregressive method (Fishman 1971) whose coverage rate is considerably higher than that reported in the original Fishman paper. The relatively comprehensive analysis in the second Adlakha and Fishman paper makes clear the attraction of using the suggested starting and stopping rules together.

2. Problem Formulation

Consider a simulation model of a queueing system with c servers in parallel, independent interarrival times with mean $1/\lambda$ and independent service times with mean $1/\omega$. Let T_i denote the elapsed time between arrivals of jobs $i-1$ and i and let S_i denote the service time of arrival i. Assume that the simulation begins with the arrival of job 1 to an empty queue and c idle servers. Let X_i denote the system time of completion i where system time denotes waiting time plus service time. Assume that an ultimate objective of analysis is to infer the characteristics of the *system time* stochastic process from a sample record of system times. Also assume that given the choice between starting data collection in an undercongested or a congested system, one prefers the congested one.

Selecting a Rule

After n completions occur during a simulation run one can estimate $1/\lambda$ and $1/\omega$ by $n^{-1} \sum_{i=1}^{n} T_i$ and $n^{-1} \sum_{i=1}^{n} S_i$ respectively. These estimates are unbiased and independent of initial conditions. Now the *activity level* or traffic intensity for a queueing system usually is defined as

$$\begin{aligned}\rho &= \text{arrival rate/no. of servers} \times \text{service rate} \\ &= \lambda/c\omega \quad ,\end{aligned}$$

for which one estimate is

$$\tilde{\rho}_n = \sum_{i=1}^{n} S_i/c \sum_{i=1}^{n} T_i \quad .$$

Since $\tilde{\rho}_n$ is usually a biased estimator of ρ an alternative, presumably more desirable, estimator is

$$\hat{\rho}_n = \frac{\rho\,\tilde{\rho}_n}{E(\tilde{\rho}_n)},$$

since $E(\tilde{\rho}_n)$ is in principle derivable for most common interarrival and service time distributions. For example, in the case of exponential interarrival and service times $E(\tilde{\rho}_n) = \rho n/(n-1)$ so that

$$\hat{\rho}_n = (n-1)\tilde{\rho}_n/n \quad .$$

Let S_1 denote the condition $|\hat{\rho}_n - \rho| \le \delta$ for $0 < \delta$ and S_2, the condition $\hat{\rho}_n > \hat{\rho}_{n-1}$. Then Fishman and Moore recommend that data collection begin with system time $T+1$ where

Rule 2 $\qquad T = \min(n:\ S_1 \text{ and } S_2 \text{ hold}).$

Condition S_1 is an accuracy requirement designed to guarantee that the sample activity level $\hat{\rho}_n$ is acceptably close to the theoretical ρ. Condition S_2 is a directional criterion designed to insure that data collection can begin only when the congestion level is increasing. Intuitively, S_2 obtains if job n has a large service time, has a small interarrival time or has both.

Using this rule on an M/M/1 queueing model Fishman and Moore

conclude:

> On the basis of the accumulated empirical evidence to date, one inclines to recommend the use of rule 2 with $\delta = .0025$. Although we do not quarrel with this recommendation, this advice should be regarded as a temporary measure on at least three grounds. Firstly, we have no experience with $\rho > .9$. Secondly, we have no experience with multi-server systems. Thirdly, the sample quantiles of starting time for rule 2 and $\delta = .9$ in ... are cause for concern. ... although 90 percent of the starting times are less than 3099 , one percent exceeds 48334 . In our opinion the risk of excessive cost is far too great to regard rule 2 as an end in itself.

As a candidate for improved performance, they suggest beginning data collection with system time $T+1$ where

Rule 1 $\qquad T = \min(n:\ S_1^*, S_2^* \text{ and } S_3^* \text{ hold}).$

$$S_1^*:\ |\hat{\rho}_{I,n} - \rho| \le \delta$$

$$S_2^*:\ \hat{\rho}_{I,n} > \hat{\rho}_{I,n-1}$$

$$S_3^*:\ mI \ne n-1$$

$$m = \text{integer} > 0\ , \qquad I = \lfloor (n-1)/m \rfloor \quad .$$

In particular for exponential interarrival and service times one has

$$\hat{\rho}_{I,n} = \frac{\sum\limits_{j=mI+1}^{n} S_j}{c \cdot \sum\limits_{j=mI+1}^{n} T_j} \cdot \frac{n - mI - 1}{n - mI} \qquad (1) .$$

In words this rule requires one to use a sample activity level based on at most m past completions. The quantity I denotes the number of times one needs to reset $\hat{\rho}_{I,N}$; *i.e.*, the number of iterations minus one. A little thought shows that

$$\mathrm{pr}(I = i) = (1 - q_m)^i \, q_m \qquad i = 0,1,\ldots$$

where q_m is the probability of success on a given iteration. Then I has a geometric distribution with mean $(1 - q_m)/q_m$ and variance $(1 - q_m)/q_m^2$. Also, the mean number of completions $E(T)$ required to meet Rule 1 satisfies

$$m(1 - q_m)/q_m < E(T) \le m/q_m \quad .$$

Now a user may choose m to suit one's convenience. However, from the viewpoint of optimality, one prefers the m that minimizes m/q_m . If several m's lead to identical minima, then one prefers the largest among them since this minimizes $\mathrm{var}(mI) = (1 - q_m)(m/q_m)^2$. In the next section we examine the performance of Rule 1 for selected values of δ , m and ρ on simulations of the M/M/1, M/M/2 and M/M/4 queueing systems.

3. Experimental Design and Method of Evaluation

To study Rule 1, we used a simulation of the M/M/c queueing system with the experimental design:

$$\rho = .7,\ .8,\ .9,\ .95$$

$$c = 1,\ 2,\ 4$$

$$\delta = .001,\ .0001$$

$$m = 1000,\ 5000 \ .$$

To reiterate, ρ denotes the activity level; c, the number of servers; δ, the tolerance to be achieved before commencing data collection; and m, the point at which an iteration terminates.

For all simulation runs the arrival rate was fixed at unity and the mean service time was taken as $c \times \rho$. For each of the $4 \times 3 \times 2 \times 2 = 48$ experiments, 1000 independent replications were performed. Independence among replications was achieved by using the last seeds of random number streams for a replication as the initial seeds for the corresponding random number streams in the next replication. On each replication of each design point, the recorded data were:

T = starting time (number of observations to satisfy the rule)

I = number of iterations minus one

X_{T+1} = system time (queueing time + service time) of the (T+1)st customer

$\hat{\rho}_{I,T}$ = activity level when the rule is satisfied (computed using (1)).

Method of Evaluation

Let X denote a system time drawn from the steady-state distribution of system time for the M/M/c queueing system. As discussed in the Fishman and Moore paper, X_{T+1} comes from a congested system if X_{T+1} stochastically dominates X. One says that the random variable W with cumulative distribution function (c.d.f.) F_W stochastically dominates the random variable V with c.d.f. F_V if $F_W^{-1}(u) - F_V^{-1}(u)$ is non-negative and not identically zero on the open interval $0 < u < 1$ where F_W^{-1} and F_V^{-1} denote the right continuous inverses of F_W and F_V respectively.

The test procedures to be used relate to the hypotheses:

H_0: X_{T+1} comes from the steady-state distribution of system time X .

H_1: X_{T+1} stochastically dominates X .

Let T_j denote the starting time on replication j and X_{j,T_j+1} the system time of completion T_j+1 on replication j . Consider J replications with sample data $X_{1,T_1+1}, \ldots, X_{J,T_J+1}$ which are independent and identically distributed when using Rule 1 . Under H_0: $X_{j,T_{j+1}}$ has the c.d.f.

$$F(x) = 1 - e^{-(\omega-\lambda)x} \qquad \text{for M/M/1}$$

$$F(x) = 1 + \frac{(1-\rho)}{(1+\rho)(2\rho-1)} e^{-\omega x} - \frac{2\rho^2}{(1+\rho)(2\rho-1)} e^{-(2\omega-\lambda)x} \qquad \text{for M/M/2}$$

and

$$F(x) = 1 + \frac{y - (4\rho-3)}{4\rho - 3} e^{-\omega x} - \frac{y}{4\rho - 3} e^{-(4\omega-\lambda)x} \qquad \text{for M/M/4}$$

where

$$y = \frac{32\rho^4}{3+9\rho+12\rho^2+8\rho^3} .$$

Table 1 presents the corresponding means, variances and coefficients of variation. Moreover,

$$Y_j = F^{-1}(X_{j,T_j+1})$$

has the uniform distribution

$$G(y) = y \qquad 0 \le y \le 1 .$$

Table 1
Parameters of the M/M/c Queue with Arrival Rate 1.0

System Time Parameters	c	ρ = .7	.8	.9	.95
mean (μ)	1	2.3333	4.0000	9.0000	19.0000
	2	2.7451	4.4444	9.4737	19.4872
	4	3.8002	5.5857	10.6898	20.7370
variance (σ^2)	1	5.4444	16.0000	81.0000	361.0000
	2	6.4278	17.2247	82.4809	362.6139
	4	11.5072	23.5341	90.3110	371.1839
coefficient of variation ($\nu = \sigma/\mu$)	1	1.0000	1.0000	1.0000	1.0000
	2	.924	.934	.959	.977
	4	.893	.870	.889	.929

The empirical c.d.f. of the Y_j's is

$$G_J(y) = \frac{1}{J} \sum_{j=1}^{J} I_{(0,y]}(Y_j) \qquad 0 \le y \le 1$$

where

$$I_{(0,y]}(Y_j) = 1 \qquad \text{if} \quad 0 < Y_j \le y$$

$$= 0 \qquad \text{otherwise.}$$

To test H_0 against all alternative hypotheses, one examines the deviations

$$\Delta(y) = G_J(y) - y \qquad 0 \le y \le 1$$

or functions of these deviations. For the Kolmogorov-Smirnov (KS) goodness-of-fit test one uses the statistic

$$D = \sup_y |\Delta(y)|$$

for which critical values appear in Miller (1953) and Owen (1962). A second test procedure uses the chi-squared test statistic

$$\chi^2 = JK \sum_{i=1}^{K} [\Delta(i/K) - \Delta(i/K - 1/K)]$$

which for integer K and large J has the chi-squared distribution with $K - 1$ degrees of freedom under H_0. A third test procedure uses the Anderson-Darling (AD) test statistic

$$W^2 = J \int_0^1 \{[\Delta(y)]^2 / y(1-y)\} \, dy$$

which is particularly sensitive to departures of $G_J(y)$ from $G(y)$ in the tails of the distribution. Critical values of the test statistic appear in Lewis (1961).

To test H_1 against H_0 one can use the KS test statistic

$$D^- = -\inf_y \Delta(y) \quad .$$

Dwass (1958) describes an additional helpful measure of discrimination. The statistic

$$U = \int_0^1 I_{[-\infty,0)}(\Delta(y)) \, dy$$

gives the proportion of $G_J(y)$ that lies below $G(y) = y$. Under H_0 U has the uniform distribution on $(0,1)$. If H_1 is true one expects U to be close to unity.

4. Evaluation

Table 2 shows the D, χ^2 and W^2 statistics for H_0 against all possible alternatives. The data show that a large number of statistics are significant at the five percent level. Before we draw any inferences, it is appropriate to consider the issue of *multiplicity*. By multiplicity we mean that when one obtains independent observations of several test statistics using, for example, the five percent significance level, then one would expect that five percent of the statistics may be significant when H_0 is true. However, the number of significant statistics in Table 2 is much larger than one would expect from random variation under H_0, and is evidence that the occurrence of these significant statistics cannot be explained satisfactorily by multiplicity.

The data in the table provide substantial evidence to reject H_0 at each design point for $\rho \leq .9$. Notice that for $\rho = .95$ all values of D, χ^2 and W^2 exceed the corresponding critical values at each design point with the exception of design points with parameters $\delta = .0001$ and $m = 5000$. This shows that H_0 is rejected for $\rho = .95$ and each c, except in the case of $\delta = .0001$ and $m = 5000$, for which the general pattern tends to favor H_0. We discuss this point shortly.

Recall that our primary objective is not merely to detect a departure from the steady-state distribution (reject H_0). In particular, we want to check for a specific type of departure from the steady state, the stochastic dominance of X_{T+1} over X. For this purpose, we use the D^- and U statistics. The statistic U gives the proportion of the empirical c.d.f. under the theoretical c.d.f. (prevalence of stochastic dominance) and the statistic D^- gives the magnitude of maximum deviation. Table 3 presents the D^- and U statistics for H_0 versus H_1. The main

Table 2

Goodness-of-Fit Statistics for Testing H_0

c	δ	m	$D(.0430)^a$				$\chi^2(30.1)^b$				$W^2(2.492)^a$			
			ρ				ρ				ρ			
			.7	.8	.9	.95	.7	.8	.9	.95	.7	.8	.9	.95
1	.001	1000	.2011*	.1348*	.0664*	.1375*	238.6*	106.2*	43.5*	136.1*	103.47*	45.06*	8.67*	36.18*
		5000	.2021*	.1475*	.0522*	.0960*	294.3*	139.9*	34.3*	74.6*	124.35*	51.92*	5.15*	22.80*
	.0001	1000	.1428*	.1451*	.0810*	.1257*	105.6*	111.9*	63.6*	148.5*	44.26*	45.57*	10.91*	28.23*
		5000	.1507*	.1505*	.0757*	.0333	175.4*	156.6*	54.0*	27.2	70.51*	62.39*	15.49*	1.75
2	.001	1000	.1243*	.1166*	.0612*	.1479*	100.0*	83.4*	59.7*	137.8*	35.88*	33.15*	6.60*	46.52*
		5000	.1353*	.1596*	.0683*	.1078*	110.6*	139.0*	28.0	86.9*	41.35*	53.51*	5.48*	31.47*
	.0001	1000	.1112*	.0810*	.0461*	.1062*	65.6*	45.4*	45.8*	118.8*	22.90*	13.23*	5.95*	20.89*
		5000	.1196*	.1165*	.0804*	.0297	96.8*	107.4*	45.7*	34.0*	34.98*	34.71*	7.79*	1.59
4	.001	1000	.1080*	.0980*	.0690*	.1122*	68.8*	62.2*	67.1*	114.6*	25.61*	17.31*	7.00*	25.11*
		5000	.1208*	.0973*	.0986*	.0848*	78.6*	67.1*	49.9*	63.8*	31.84*	20.22*	18.96*	11.37*
	.0001	1000	.0584*	.0614*	.0518*	.1164*	33.0*	30.8*	71.0*	117.2*	3.26*	7.34*	5.42*	22.75*
		5000	.0980*	.1314*	.0619*	.0423	57.8*	177.3*	28.1	34.4*	21.88*	57.94*	7.92*	6.20*

[a] Critical value at the 5% level.

[b] Critical value for χ^2 at the 5% level with 19 degrees of freedom.

*Significant at the 5% level.

Table 3

Testing H_0 versus H_1

			$D^-(.0381)^a$				$U(.95)^a$			
			ρ				ρ			
c	δ	m	.7	.8	.9	.95	.7	.8	.9	.95
1	.001	1000	.2011*	.1384*	.0664*	.0159	.9983*	.9940*	.7576	.1213
		5000	.2021*	.1475*	.0522*	.0064	.9999*	.9976*	.9859*	.0425
	.0001	1000	.1428*	.1451*	.0810*	.0368	.9984*	.9994*	.7723	.3278
		5000	.1507*	.1505*	.0757*	.0333	.9917*	.9940*	.9953*	.8229
2	.001	1000	.1243*	.1166*	.0612*	.0023	.9996	.9889*	.7516	.0335
		5000	.1353*	.1596*	.0683*	.0000	.9931*	.9934*	.8216	.0002
	.0001	1000	.1112*	.0810*	.0380	.0569*†	.9946*	.9191*	.5749	.3597
		5000	.1196*	.1165*	.0804*	.0297	.9998*	.9974*	.9728*	.5999
4	.001	1000	.1080*	.0980*	.0690*	.0224	.9920*	.9276*	.7294	.2802
		5000	.1208*	.0973*	.0986*	.0023	.9838*	.9870*	.9868*	.0076
	.0001	1000	.0584*	.0614*	.0518*	.0299	.8030	.9915*	.7104	.4025
		5000	.0980*	.1314*	.0619*	.0014	.9983*	.9980*	.9715*	.0095

[a]Critical Value at the 5% level.

*Significant at the 5% level.

†The statistic D^+ is .1062 at this design point.

observations are:

(1) For $\rho = .7, .8$ both the D^- and the U statistics support the hypothesis of stochastic dominance of X_{T+1} over X at all design points, with the exception of the U statistic at one design point ($\rho = .7, c = 4, \delta = .0001$ and $m = 1000$). Also observe that the negative deviation D^- is reduced as δ decreases or as c increases.

(2) For $\rho = .9$ the D^- statistics support the hypothesis of stochastic dominance at all design points, but the U statistics generally show this behavior only with $m = 5000$.

(3) For $\rho = .95$ both the D^- and the U statistics fail to support the hypothesis of stochastic dominance at all design points.

Although Rule 1 fails to achieve stochastic dominance for $\rho = .95$, the statistics in Table 2 generally favor H_0 for $\rho = .95$ with $\delta = .0001$ and $m = 5000$. Here we consider the possibility that X stochastically dominates X_{T+1}, implying that X_{T+1} comes from an undercongested system. Formally, we consider for $\rho = .95$ the alternative hypothesis

H_2: X stochastically dominates X_{T+1}.

To test H_0 versus H_2 for $\rho = .95$, we use the KS statistic

$$D^+ = \sup_y \Delta(y) \quad .$$

The results appear in Table 4 . The data support H_2 for all design points with the exception of design points (ρ = .95, c = 1, δ = .0001, m = 5000) and (ρ = .95, c = 2, δ= .0001, m = 5000) . At these two design points the steady state is supported. The selection of H_0 over H_2 in these two cases is reassuring, for if H_0 is more credible than H_1 , we prefer that H_0 also be more credible than H_2 . Achieving the steady state is acceptable, because this assures that the initial conditions of the simulation no longer play a role.

The empirical evidence suggests that the parameters δ = .0001 and m = 5000 give the most satisfactory results over the entire range

Table 4

Testing H_0 versus H_2 for ρ= .95

		$D^+(.0381)^a$		
		c		
δ	m	1	2	4
.001	1000	.1375*	.1479*	.1122*
	5000	.0960*	.1078*	.0848*
.0001	1000	.1257*	.1062*	.1164*
	5000	.0261	.0269	.0423*

[a]Critical value at the 5% level.

*Significant at the 5% level.

of activity level ρ considered in this study [.7, .95] . For $\rho \leq .9$, Rule 1 with these parameter values appears to induce stochastic dominance. Though the test statistics for $\rho = .95$ fail to support the hypothesis of stochastic dominance, the steady state seems to be achieved for $c = 1,2$. As stated previously, this is acceptable. Therefore, we recommend the values of $\delta = .0001$ and $m = 5000$ for use with Rule 1 .

Since data collection starts with the system time X_{T+1} , it is of interest to study the bias and the dispersion of X_{T+1} . Tables 5 through 7 present the sample mean $\bar{X}_{T+1}$, the sample variance $\hat{\sigma}^2(X_{T+1})$ and the sample coefficient of variation $\hat{\nu}(X_{T+1})$ computed over the 1000 replications of each design point for the M/M/1, M/M/2 and M/M/4 queueing simulations. The data show that $\bar{X}_{T+1}$ is greater than the corresponding theoretical mean in experiments where Rule 1 achieves stochastic dominance, and is generally less than the theoretical mean in other cases. To test the statistical significance of the difference between the theoretical and sample mean for Rule 1 with our recommended parameters, $\delta = .0001$ and $m = 5000$, we consider the hypothesis

H_3: $\bar{X}_{T+1}$ has the steady-state mean

against the alternative hypothesis

H_4: $\bar{X}_{T+1}$ does not have the steady-state mean.

The test statistics appear in Table 8 . The data show that $\bar{X}_{T+1}$ is

Table 5

Sample Mean, Variance and Coefficient of Variation for System Time X_{T+1}

M/M/1

Statistic	δ	m	Activity Level ρ 0.7	0.8	0.9	0.95
$\bar{X}_{T+1}$	.001	1000	3.416	5.002	9.052	13.412
		5000	3.629	5.394	9.788	14.625
	.0001	1000	2.905	5.022	8.839	14.558
		5000	3.249	5.415	10.404	18.625
$\hat{\sigma}^2(X_{T+1})$	.001	1000	8.022	17.523	58.365	124.404
		5000	9.762	23.786	86.014	187.868
	.0001	1000	6.008	18.446	50.249	160.000
		5000	8.007	22.655	90.903	277.835
$\hat{\nu}(X_{T+1})$	.001	1000	.829	.837	.844	.832
		5000	.861	.904	.948	.937
	.0001	1000	.844	.855	.802	.869
		5000	.871	.879	.916	.895
Theoretical Quantities						
	μ		2.333	4	9	19
	σ^2		5.44	16	81	361
	ν		1	1	1	1

Table 6

Sample Mean, Variance and Coefficient of Variation for System Time X_{T+1}

M/M/2

Statistic	δ	m	Activity Level ρ			
			0.7	0.8	0.9	0.95
$\overline{X}_{T+1}$	.001	1000	3.453	5.393	9.235	13.403
		5000	3.489	5.873	10.221	14.816
	.0001	1000	3.178	4.865	8.683	15.631
		5000	3.421	5.708	10.452	18.698
$\hat{\sigma}^2(X_{T+1})$	.001	1000	8.651	18.971	52.532	138.434
		5000	8.236	23.631	85.478	212.971
	.0001	1000	6.692	15.314	49.372	165.322
		5000	8.244	26.280	86.144	275.240
$\hat{\nu}(X_{T+1})$	.001	1000	.852	.808	.785	.878
		5000	.823	.832	.905	.985
	.0001	1000	.814	.804	.809	.823
		5000	.839	.898	.888	.887
Theoretical Quantities						
		μ	2.7451	4.4444	9.4737	19.4872
		σ^2	6.4278	17.2247	82.4809	362.6139
		ν	.924	.934	.959	.977

Table 7

Sample Mean, Variance and Coefficient of Variation for System Time X_{T+1}

M/M/4

Statistic	δ	m	Activity Level ρ 0.7	0.8	0.9	0.95
$\overline{X}_{T+1}$	.001	1000	4.486	6.309	10.407	16.105
		5000	4.575	6.439	12.244	17.411
	.0001	1000	3.826	6.050	10.122	16.712
		5000	4.498	7.570	11.773	18.141
$\hat{\sigma}^2(X_{T+1})$	.001	1000	12.842	23.458	58.120	150.670
		5000	13.153	24.898	94.514	225.918
	.0001	1000	9.356	24.461	56.175	176.487
		5000	13.759	40.273	95.986	237.959
$\hat{\nu}(X_{T+1})$	.001	1000	.799	.768	.733	.762
		5000	.792	.775	.794	.863
	.0001	1000	.799	.818	.740	.795
		5000	.825	.838	.832	.850
Theoretical Quantities						
	μ		3.8002	5.5857	10.6898	20.7370
	σ^2		11.5072	23.6341	90.3110	371.1839
	ν		.893	.870	.889	.929

Table 8

Student t Statistics[a] for $\overline{X}_{T+1}$

$\delta = .0001$ and $m = 5000$

c \ ρ	.7	.8	.9	.95
1	10.237*	9.401*	4.657*	-.711
2	7.444*	7.797*	3.333*	-1.504
4	5.959*	9.888*	3.496*	-5.322*

[a]The entries are $(\overline{X}_{T+1} - \mu)\sqrt{1000} \,/\, \hat{\sigma}(X_{T+1})$, the critical value at the 5% level is 1.96 .

*Significant at the 5% level.

significantly biased upward for $\rho \le .9$. This is consistent with the stochastic dominance established for $\rho \le .9$. For $\rho = .95$ and $c = 4$ a significantly downward-biased condition for $\overline{X}_{T+1}$ is indicated.

The data in Tables 5 through 7 show that for each design point, $\hat{\nu}(X_{T+1})$ is less than its theoretical value. This observation is of importance because it indicates the probability of getting an X_{T+1} from the tail distribution is not increased over the corresponding steady-state probability and, therefore, the upward bias in $\overline{X}_{T+1}$ is not the result of including a disproportionate number of extreme waiting times. Figure 1 presents the sample cumulative distribution function of X_{T+1} and the c.d.f. of X for the case of $\rho = .9$, $c = 1$, $\delta = .0001$ and $m = 5000$. The figure explains the fact that Rule 1 concentrates the probability of system time away from the tails.

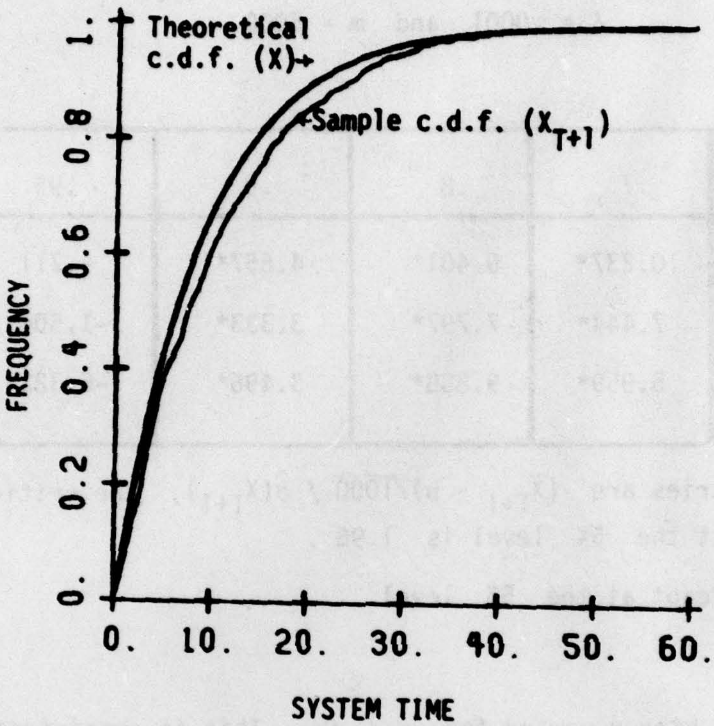

Figure 1
c.d.f.'s of X_{T+1} and X
ρ = .9, c = 1, δ = .0001, m = 5000

A basic assumption inherent in the development of Rule 1 is that the correlation between X_{T+1} and T becomes small as δ decreases. This was first observed in Fishman and Moore (1978) and was one of the motivating factors in considering Rule 1 . In principle, one would like X_{T+1} and T to be independent. Then there would be no need for concern that a small T on a particular run would produce an undercongested system for starting data collection. Table 9 presents the sample

correlation between X_{T+1} and T for each design point. Notice that for given m, there is a tendency for the correlation to diminish as δ decreases, albeit exceptions exist.

Under the null hypothesis: $\mathrm{corr}(X_{T+1}, T) = 0$, the sample correlation coefficient asymptotically has the normal distribution with mean zero and variance $1/J$ for J independent replications. Let us concentrate on the recommended design parameters $\delta = .0001$ and $m = 5000$. Significance occurs at the five percent level at $\rho = .9$ for M/M/1 and M/M/4 and at $\rho = .95$ for M/M/1 and M/M/2. Although room for improvement exists for creating a rule that makes X_{T+1} and T independent, the relatively small magnitudes of the significant correlations encourages us to recommend Rule 1 at present with minimal concern.

In a study involving four factors ρ, c, δ and m, it is of interest to see how performance is affected by different levels of the factors. One way to measure performance with regard to stochastic dominance is to observe how D^- and U change with the alternative levels. An analysis of variance (ANOVA) enabled us to investigate these questions. To bring the data in closer conformity with the assumptions of ANOVA, D^- and U were transformed to standardized normal variates under the assumption that H_0 was true. For example, under H_0 $\Phi^{-1}(U)$ is a normal deviate where Φ^{-1} is the inverse function of the normal distribution.

ANOVA's were performed for the transformed D^- and U statistics separately. Because of space considerations, we report the most relevant results for fixed $\delta = .0001$ and $m = 5000$ only.[†] They show that for the

[†]Complete details of the ANOVA studies appear in Adlakha (1979).

Table 9

Sample Correlation Coefficients[a] between X_{T+1} and T

c	δ	m	Activity Level ρ			
			.7	.8	.9	.95
1	.001	1000	.084*	.072*	.076*	.282*
		5000	.059	.031	.113*	.211*
	.0001	1000	-.049	-.035	.025	.114*
		5000	.043	.038	.108*	.129*
2	.001	1000	.044	.062*	.092*	.275*
		5000	.028	.078*	.094*	.241*
	.0001	1000	.044	.027	.044	.088*
		5000	-.004	.073*	.041	.120*
4	.001	1000	-.021	.073*	.020	.176*
		5000	.013	.026	.151*	.205*
	.0001	1000	.031	.041	-.008	.166*
		5000	.010	-.036	.063*	.053

[a]The critical values with 1000 observations at the 5% and 1% significance levels are .062 and .081, respectively.

*Significant at the 5% level.

range $.7 \le \rho \le .95$ performance is relatively insensitive to c. This is a gratifying result, for it suggests a generality in the performance of Rule 1 with regard to varying the number of servers in a model. However, the analyses indicate an association between performance and ρ that reinforces the visual observations in Table 3. Although more work is needed to remove this association, we continue to recommend Rule 1 for $\rho \le .95$ because of its demonstrated success. Naturally, a rule that performs independently of ρ is the ultimate goal.

5. Distribution of Starting Time

A desirable characteristic of a starting rule is that it not require excessive amounts of computer time. The computer time required by a starting rule is essentially composed of a program set-up time plus a running time component that is generally proportional to the number of observations generated. For all practical applications the set-up time is insensitive to changes in starting rules when compared with the running time component. Therefore, it is of interest to study the starting time T associated with our starting rule, since it is this variable that determines the cost (in computer time) of the rule.

We are interested in studying the variation in the mean starting time in response to changes in the parameters ρ, c, δ and m. Adlakha (1979) contains these results for the complete experimental design. Here we focus on the distribution of T for Rule 1 with the recommended parameters $\delta = .0001$ and $m = 5000$.

We first discuss quantiles. The $100 \times p$ percent quantile of the distribution of T is $\min[n: \ pr(T \le n) = p]$. Table 10 presents the sample quantiles, which appear to be relatively insensitive to ρ and c . Although the 95 percent quantiles, which tend to be about 10,000 , are comparable to those observed by Fishman and Moore's starting Rule 2 , the higher quantiles and the maximum value have decreased drastically. This indicates that our recommended rule has substantially reduced the skewness in the distribution of T .

The sample mean, standard deviation, and coefficient of variation of T also appear in Table 10 . The data show that the mean starting time varies between 2837 and 3387 . These mean values occur around the 65 percent quantile as compared to the 90 percent quantile for Rule 2 in Fishman and Moore (1978) . The coefficient of variation of T is approximately equal to one in each case. These observations again suggest that the starting time distribution is not as skewed as the distribution obtained in the earlier work.

To see the influence of this reduction in skewness on the cost of a simulation run, we consider the case of $\rho = .9$ with the cost function

$$C(T) = c_0 + c_1 T + c_2 I_{[T^*,\infty)}(T) \quad ,$$

for example, and let $T^* = 30{,}000$. A cost function of this type arises when one runs out of computer time or allocated space during a simulation run and has to run an experiment over again. One can also conceive of such a cost

Table 10

Sample Quantiles of Starting Time for Rule 3

$\delta = .0001$, $m = 5000$

c	1				2				4			
$100 \times p$ \ ρ	.7	.8	.9	.95	.7	.8	.9	.95	.7	.8	.9	.95
1	46	50	67	68	47	53	60	69	55	54	66	35
2	82	74	101	96	92	83	104	133	79	100	97	78
5	154	149	192	198	169	169	184	235	168	167	176	169
10	252	279	283	318	282	262	303	357	284	303	287	328
15	336	363	377	452	377	398	396	483	397	436	418	423
20	415	469	491	556	478	477	520	603	511	563	529	527
25	523	574	635	680	568	585	627	746	631	710	691	659
30	649	684	763	828	679	744	761	882	778	818	833	803
35	771	800	877	1004	823	868	921	1044	906	993	1002	966
40	916	958	1062	1169	985	1016	1112	1301	1064	1172	1194	1118
45	1105	1196	1271	1360	1186	1186	1286	1516	1253	1412	1398	1370
50	1297	1366	1509	1576	1437	1464	1537	1783	1563	1737	1766	1621
55	1577	1679	1897	1926	1677	1809	1832	2166	1902	2067	2125	2006
60	1999	2068	2329	2375	1930	2238	2153	2694	2357	2109	2631	2468
65	2579	2664	2850	2906	2366	2835	2700	3489	2828	2696	3352	2955
70	3418	3384	3823	3722	3257	3613	3553	4609	3807	3450	4271	4103
75	4895	4628	5097	5146	4464	4887	4798	5489	5135	4662	5263	5184
80	5428	5382	5554	5687	5352	5406	5563	5977	5627	5448	5596	5579
85	5966	5806	6006	6393	5663	5822	6188	6640	6113	6013	6082	6215
90	6884	6683	7560	7816	6540	6752	7202	7982	7213	7150	7170	7173
95	10498	10224	10490	10609	10531	10137	10490	10877	10207	10606	10398	10542
98	13534	12013	15306	12825	12418	11581	15306	13896	13985	12524	12566	14193
99	15919	15255	18870	15916	15751	13199	17748	16762	15587	15698	14058	18091
min	9	2	11	6	18	9	33	17	12	7	2	8
max	27005	24715	34605	25886	27391	26409	31787	25516	25768	21444	21950	27454
$\bar{T}$	2906	2850	3188	3174	2839	2877	3057	3387	3062	3035	3145	3149
$\hat{\sigma}_T$ [a]	3561	3332	4043	3627	3442	3218	3682	3696	3496	3407	3337	3656
$\hat{\nu}_T$ [a]	1.23	1.17	1.27	1.14	1.21	1.12	1.20	1.14	1.12	1.14	1.06	1.16

[a] The quantities $\hat{\sigma}_T$ and $\hat{\nu}_T$ denote the sample standard deviation and the coefficient of variation respectively.

function when computer reliability is low. For Rule 2 in the Fishman and Moore paper, one has

$$E[C(T)] = c_0 + c_1E(T) + c_2 \times .02$$

and for Rule 1

$$E[C(T)] = c_0 + c_1E(T) + c_2 \times .001 \quad .$$

Since the mean starting time obtained is approximately the same with Rule 2 and the recommended Rule 1, a significant reduction in the expected cost is achieved when c_2 is much greater than c_1. Avoiding long tail starting time distributions is clearly a desirable objective.

6. Conclusions and Proposed Algorithm

The empirical evidence of this study strongly indicates that the use of Rule 1 results in the starting of data collection when a system is congested (for $\rho \leq .90$) or is at least in the steady state (for $\rho = .95$). Although a firm theoretical basis for this dilution of the influence of empty and idle initial conditions remains to be developed, we believe that the use of Rule 1 is a reasonable recommendation for a wider class of queueing simulations other than those tested. The supposition here is that one can compute the theoretical activity level exactly. The particular form that the activity level assumes is of course a function of the system being simulated.

There are several ways by which a user can implement Rule 1 in a simulation program. Essential steps are provided in algorithm START .

<u>Algorithm START</u>

Let $A \equiv$ current sample activity level

$B \equiv$ old sample activity level

$I \equiv$ number of iterations minus one

$\delta \equiv$ tolerance level (given)

$m \equiv$ iteration length (given)

$n \equiv$ number of completions .

1. Start the simulation in the empty and idle state, $n \leftarrow 0$, $I \leftarrow 0$, $B \leftarrow 0$.
2. Simulate until next completion and $n \leftarrow n+1$.
3. Compute A based on last $n - mI$ completions.

Rule 1

4. If $B \neq 0$, $|A - \rho| \leq \delta$ and $A > B$ go to step 7 .
5. If $n \neq m(I+1)$, $B \leftarrow A$, go to step 2 .
6. $I \leftarrow I+1$, $B \leftarrow 0$, go to step 2 .
7. Begin data collection at next completion.

7. References

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This paper proposes a rule for determining when to start collecting data in a queueing simulation. The rule is designed to reduce dependence between the empty (queue) and idle (servers) *initial conditions* and the collected sample record. The rule is an outgrowth of earlier work by Fishman and Moore (1978) and relies on a comparison between *a priori* information on the activity level (traffic intensity) and a corresponding sample estimate

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computed during the course of simulation. Experiments with simulations of the M/M/c queue with $c = 1,2,4$ and $\rho = .7, .8, .9, .95$ reveal that the rule reduces and in most cases removes the dependence on the empty and idle initial conditions. In particular, the rule begins data collection when the simulation is in a congested state or in the steady state. The rule is well behaved in that it has low probabilities of requiring long runs before data collection is started. Although our data suggest an association between the rule's performance and activity level, the performance is insensitive to variation in the number of servers. Since the rule is based upon the activity level, a parameter that frequently can be computed from the input parameters of the simulation, the rule is easily generalized to a wider class of queueing simulations. A subsequent study (Adlakha and Fishman 1979) demonstrates the appeal of the starting rule when used with a proposed stopping rule for computing interval estimates of parameters of interest.